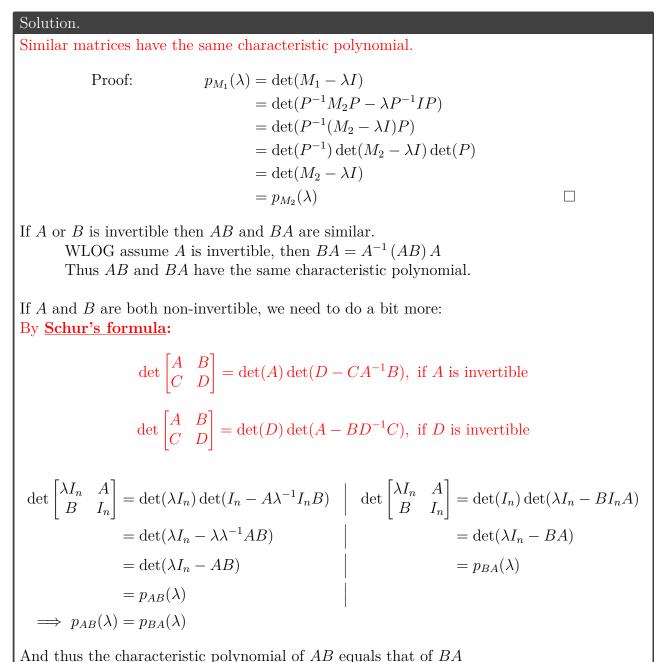
## Rutgers University: Algebra Written Qualifying Exam January 2018: Problem 3 Solution

## Exercise.

(a) Prove that for any square matrices A and B of size n with coefficients in some field the characteristic polynomial of AB equals that of BA.



(b) Give an example of square matrices A and B such that the minimal polynomial of AB does not equal that of BA.

Solution.		
$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	and	$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
$\implies AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	and	$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
$\implies q_{BA}(x) = x$	BUT	$q_{AB} \neq x$ , since $AB \neq 0$